Conics - Hyperbolas

Warmup

Find the foci

1. $\dfrac{(x+6)^2}{36} - \dfrac{(y+3)^2}{9} = 1$

$(-6 \pm 3\sqrt{5}, -3)$

2. $\dfrac{(y-3)^2}{25} - \dfrac{(x-2)^2}{16} = 1$

$(2, 3 \pm \sqrt{41})$

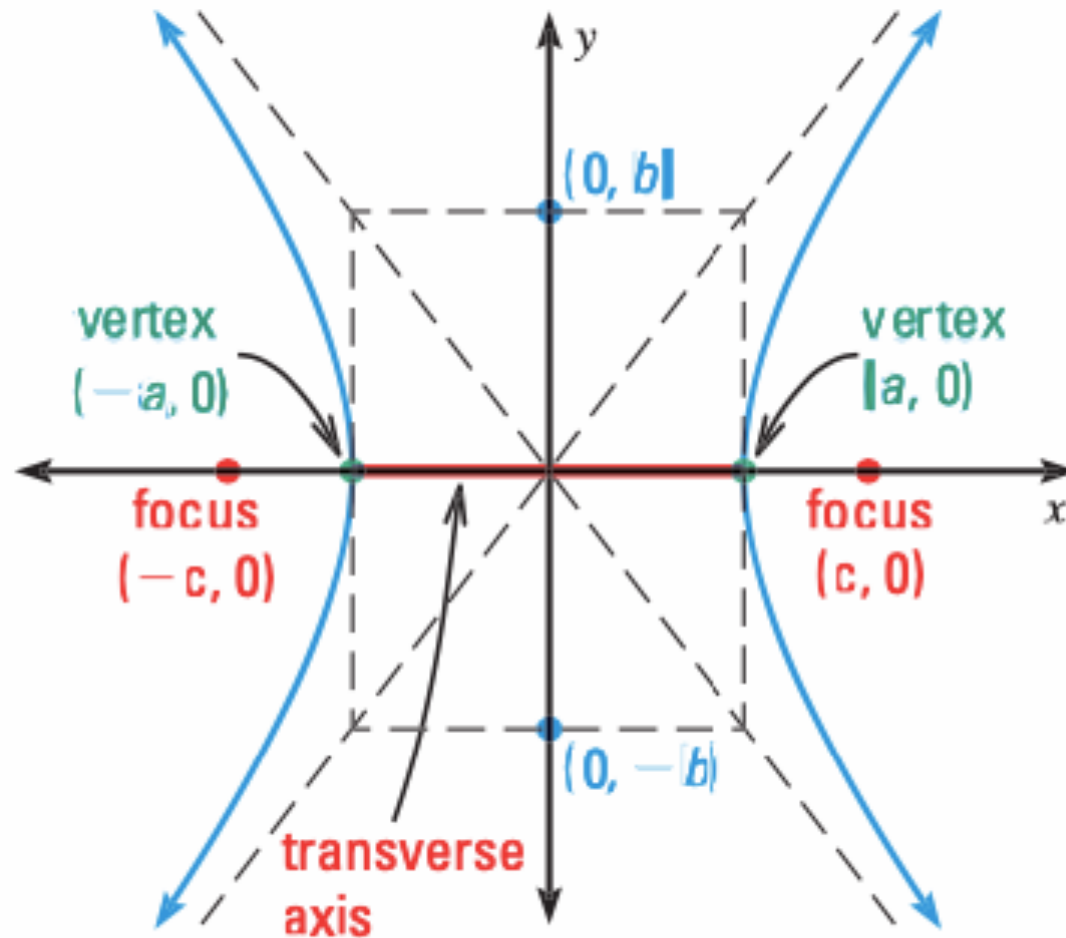3. $\dfrac{(y-4)^2}{16} - \dfrac{(x+2)^2}{9} = 1$

$(-2, 4 \pm 5)$

4. $\dfrac{(x+1)^2}{4} - \dfrac{(y-4)^2}{9} = 1$

$(-1 \pm \sqrt{13}, 4)$

Conics - Hyperbola

Definition

vertex $(-a, 0)$

vertex $(a, 0)$

$(0, b)$

focus $(-c, 0)$

focus $(c, 0)$

$(0, -b)$

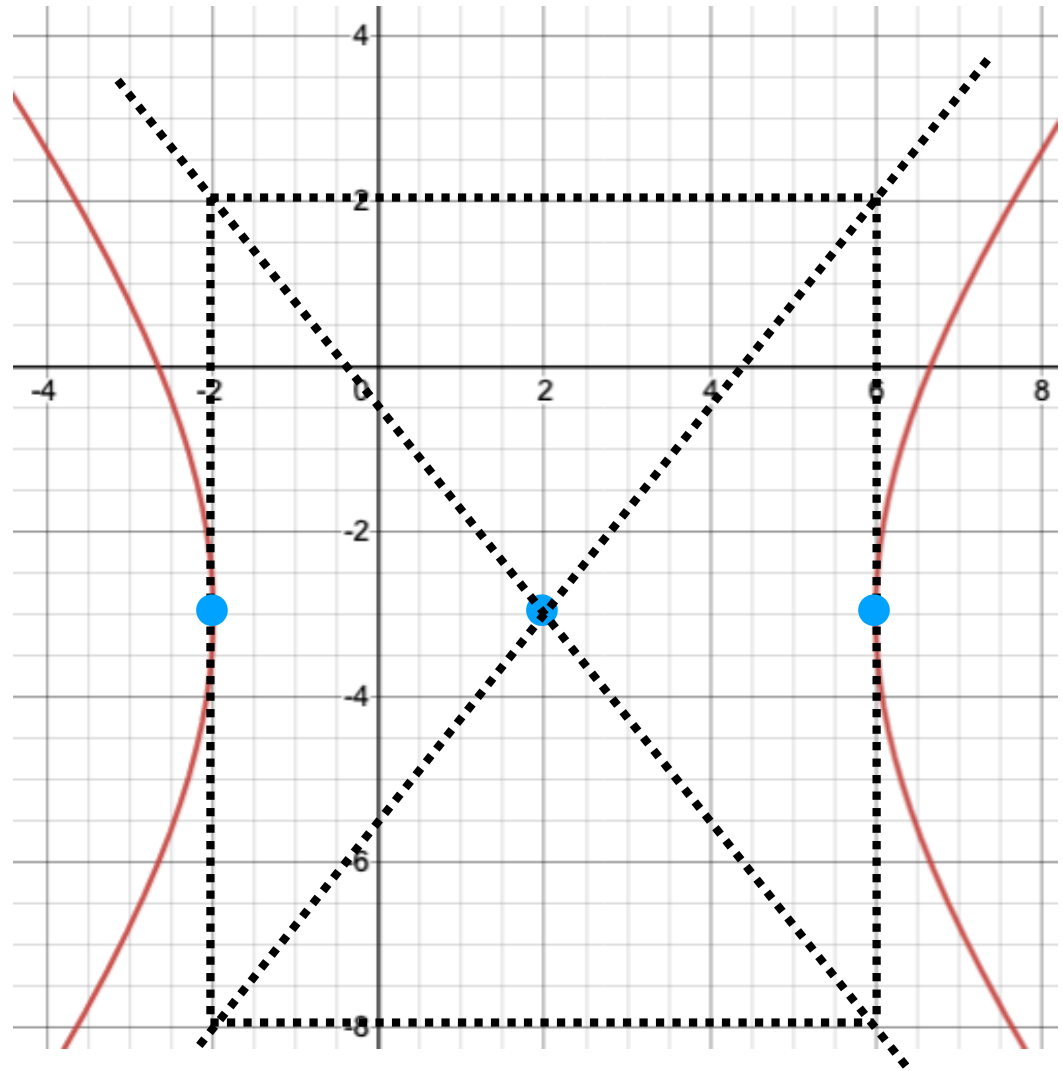**transverse axis**

Hyperbola with horizontal transverse axis

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Conics - Hyperbolas

Graph and find the foci.

$$\frac{(x-2)^2}{16} - \frac{(y+3)^2}{25} = 1$$

$$(2 \pm \sqrt{41}, -3)$$

Graph the following

$$9x^2 + 18x - 4y^2 + 16y - 43 = 0$$

Center: $(-1, 2)$

Foci: $(-1 \pm \sqrt{13}, 2)$

Asymptotes: $y = \dfrac{3}{2}x + \dfrac{7}{2}$

$y = -\dfrac{3}{2}x + \dfrac{1}{2}$

Conics - Hyperbolas

Find the equation with the given.

Asymptotes: $y = \dfrac{5}{2}x - 7$

$y = -\dfrac{5}{2}x + 3$

Opens vertically

a = 10

$$\dfrac{(y+2)^2}{100} - \dfrac{(x-2)^2}{16} = 1$$

Logarithm Property

$$\frac{1}{\log_x y} = \frac{1}{\frac{\log y}{\log x}} = \frac{\log x}{\log y} = \log_y x$$

$$\frac{1}{\log_5 10} = \log_{10} 5 \qquad \log_2 7 = \frac{1}{\log_7 2}$$

6.5 - Properties of Logarithms

Changing bases

$$y = 9^{\log_2 81}$$

$$\log_9 y = \log_2 81$$

$$\frac{\log_2 y}{\log_2 9} = \log_2 81$$

$$\log_2 y = \log_2 9 \cdot \log_2 81$$

$$\log_2 y = \log_2 81^{\log_2 9}$$

$$y = 81^{\log_2 9}$$

$$y = 4^{\log_2 x}$$

$$y = x^{\log_2 4}$$

$$y = x^2$$

6.6 - Solving Exponential and Logarithmic Equations

Find the inverse

$$y = \left(\frac{2^x + 6}{-3}\right)^{1/3} \qquad y = \log_2(-3x^3 - 6)$$

Practice

1. $y = \left(\frac{e^x + 10}{2}\right)^{1/5}$

2. $y = \log_5(-4x + 6) + 4$

$$y = \ln(2x^5 - 10) \qquad y = -\frac{5^{x-4} - 6}{4}$$

A substance decays 22% each day. After 7 days, there are 9 milligrams of the substance remaining. How many milligrams were there initially?

$$51.24 \; mg$$

Anil opened a bank account with an interest rate of 4.8% that is compounded annually. He invested $3,890 in the account at the beginning of 1999 but had to make a withdrawal from his account at the end of 2007 in the amount of $2,300 with no penalty. How much money is in his account at the end of 2016?

$$\$5,538.57$$

A new Toyota Camary costs $36,000 and decreases in value 20% per year. How old is the car (in years) when it is worth half its original cost?

$3.1 \; yrs$

Carbon-14 has a half-life of approximately 6,000 years. A sample, originally 100 kg of the material, has decayed through the years and now only 6.25 kg remains. Approximately how old is the sample?

$24,000 \; yrs$

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